Optimal Derotation of Shared Acceleration Time Series by Determining Relative Spatial Alignment*

Rene Mayrhofer
Institute of Networks and Security, Johannes Kepler University Linz
rene.mayrhofer@jku.at

Helmut Hlavacs
University of Vienna
helmut.hlavacs@univie.ac.at

Rainhard Dieter Findling
JRC u'smile, University of Applied Sciences Upper Austria
rainhard.findling@fh-hagenberg.at

August 31, 2015

Abstract

Type: **Research paper**

Purpose Detecting if two or multiple devices are moved together is an interesting problem for different applications. However, these devices may be aligned arbitrarily with regards to each other, and the three dimensions sampled by their respective local accelerometers can therefore not be directly compared. The typical approach is to ignore all angular components and only compare overall acceleration magnitudes — with the obvious disadvantage of discarding potentially useful information.

Approach In this paper, we contribute a method to analytically determine relative spatial alignment of two devices based on their acceleration time series. Our method uses quaternions to compute the optimal rotation with regards to minimizing the mean squared error.

Practical implications After derotaion, the reference system of one device can be (locally and independently) aligned with the other, and thus that all three dimensions can consequently be compared for more accurate classification.

*This article is an extended version of a conference paper previously published in iiWAS 2014 (16th International Conference on Information Integration and Web-based Applications & Services) by ACM Press.

Findings Based on real-world experimental data from smart phones and smart watches shaken together, we demonstrate the effectiveness of our method with a magnitude squared coherence metric, for which we show an improved EER of 0.16 (when using derotation) over an EER of 0.18 (when not using derotation).

Originality Without derotating time series, angular information cannot be used for deciding if devices have been moved together. To the best of our knowledge, this is the first analytic approach to find the optimal derotation of the coordinate systems, given only the two 3D time acceleration series of devices (supposedly) moved together. It can be used as the basis for further research on improved classification towards acceleration-based device pairing.

Keywords mobile devices, acceleration time series, quaternion derotation, device authentication

1 Introduction

Common movement can be detected from sufficiently similar acceleration sensor data and has interesting applications in mobile and ubiquitous computing. This includes determining if devices are carried by the same user (Lester et al., 2004) or transported on the same vehicle (Marin-Perianu et al., 2007) as well as an interaction method for securely pairing handheld devices (Mayrhofer and Gellersen, 2009; Bichler et al., 2007; Kirovski et al., 2007). However, such common movement is inherently three-dimensional. In the general case, the relative alignment of two (or multiple) accelerometers embedded in different devices is unknown: similar devices may be rotated arbitrarily with regards to each other and different devices may embed their accelerometers with arbitrary orientations. Therefore, the three dimensions sampled independently will typically not be aligned and are therefore not directly comparable.

A standard approach to deal with this issue is to discard all angular (i.e. directional) information from the 3D vectors and only use their magnitude (i.e. the length of each vector computed in an Euclidean space). This reduces three dimensions to a single one that is invariant concerning orientation. Even when two co-located accelerometers are oriented differently, they will experience similar overall acceleration magnitudes. However, this simple approach discards potentially valuable information that could be helpful in determining if accelerations are sufficiently similar to each other or not (cf. Section 3).

In this article, we describe a method to explicitly determine the relative alignment of two mobile devices with regards to each other based on their recorded acceleration time series. The underlying assumption is that both devices are moved (relatively closely) together and therefore share sensor readings that are only offset by 3D rotation but otherwise similar. Specifically, we assume that both devices experienced similar translation and rotation with regards to a common reference system. Our approach uses quaternions (Section 4) to analytically compute optimal rotation between both device reference systems (Section 5) and, based on real-world sensor data, works even in the presence of small distances between the devices and typical sensor noise (Section 6).

2 Related Work

Known applications of common movement presented in the context of mobile and pervasive computing (e.g. (Lester et al., 2004; Marin-Perianu et al., 2007; Mayrhofer and Gellersen, 2009; Bichler et al., 2007; Kirovski et al., 2007; Groza and Mayrhofer, 2012)) have so far taken the simple approach and discarded angular information. However, we suggest that all of these could benefit to various degrees from taking this information into account. Especially when used for securing device communication (Mayrhofer and Gellersen, 2009; Bichler et al., 2007; Groza and Mayrhofer, 2012) would this be valuable — any information that is shared between the legitimate devices but not directly available to a potential attacker increases the latter's entropy of the resultant cryptographic key and consequently improves the security level.

Kunze and Lukowicz (2008) have suggested position-invariant heuristics for dealing with sensor displacement to improve movement recognition with a single sensor (accelerometer and/or gyroscope). Our approach complements this work when multiple sensors are in use, e.g. to detect if a mobile phone and a wrist watch describe the same movement and are therefore on the same hand.

Quaternions have been used to minimize the root-mean-squared deviation (RMSD) between solid bodies (Coutsias et al., 2004). We build upon this work by translating it from body rotation to determining the relative alignment of 3D acceleration time series. Another related use of quaternions is representing orientations in hand and head movement (Choe and Faraway, 2004).

3 Problem Overview

Determining if two (or multiple) devices are moved together based on their respective local accelerations can be seen as a classification problem. When they are moved together, sensor noise and systematic error will still lead to (slightly) different sensor time series. When they are moved separately (but for example with similar frequency and amplitude components), they might still be "close" for some similarity measure (cf. (Mayrhofer and Gellersen, 2009) for experimental "positive" and "negative" data). The systematic error is intrinsic: even if the devices are held perfectly together and do not move with regards to each other *and* the sensors are perfect and do not exhibit any sampling noise at all, there will still be differences in acceleration time series whenever rotation is part of the shared movement. This is because of different centers, i.e., the physical placement of the respective accelerometers. Think of one accelerometer on the outer curve and the other on the inner curve of a common rotation; they move together without relative movement, but take different paths in 3D that consequently lead to different *local* accelerations.

This issue is independent of the chosen similarity measure and also occurs when only using the magnitude. In fact, discarding angular information makes it even harder to determine that the devices were moved together because locally measured rotational components would in this case be similar, while the magnitudes differ. Recently suggested heuristics explicitly discard accelerometer time series in periods of large rotational movement (Kunze and Lukowicz, 2008). We expect classification accuracy to improve noticeably when comparing three dimensions instead of one throughout various different use cases.

Our approach to retain this 3D information is for devices to – locally and independently of each other – align the two coordinate systems for the subsequent comparison. We analytically determine the "optimal" rotation between these coordinate systems given only the two 3D time series (which are for example exchanged securely using an interlock protocol and session keys as described previously by Mayrhofer and Gellersen (2009)) and the assumption of shared movement. In the scope of this paper, we define optimal to minimize the mean squared error between all of the sample points. One method to analytically determine the relative orientation is to use quaternions.

4 Quaternions and Rotations

Quaternions can be used to represent rotations in a three dimensional space. Furthermore, they possess favorable properties like avoiding so-called "gimbal locks" or enabling easy interpolation, something that other approaches like Euler angles and matrix-based rotation do not exhibit. It is thus straight forward to use quaternions to find the optimal rotation between two sets of vectors.

In the following, only the most important aspects of quaternions are presented, and we follow the notation of (Coutsias et al., 2004). More details about quaternions can be found, e.g. in (Kuipers, 2002). A quaternion is a tuple $q = (q_0, \mathbf{q})$, with $\mathbf{q} = (q_1, q_2, q_3)'$. Note that like in Matlab/Octave the operator " $'$ " denotes transposition, and all vectors without it are column vectors. Quaternions are essentially a generalization of complex numbers, i.e. a quaternion consists of a real part (q_0) and three imaginary parts (q_1, q_2, q_3). In the area of three dimensional spaces, this imaginary part may take over the part of a 3D-vector. Since quaternions form up an algebraic structure called *division ring*, they allow the algebraic operations addition and multiplication, which for $a = (a_0, \mathbf{a})$ and $b = (b_0, \mathbf{b})$ are defined as follows:

$$\begin{aligned} a + b &= (a_0 + b_0, \mathbf{a} + \mathbf{b}) \\ ab &= (a_0 b_0 - \mathbf{a} \cdot \mathbf{b}, a_0 \mathbf{b} + b_0 \mathbf{a} + \mathbf{a} \times \mathbf{b}). \end{aligned} \tag{1}$$

Here $\cdot$ and $\times$ denote the standard dot and cross products known from Euclidean vector spaces. Interestingly, multiplication is associative, but not commutative, i.e. in general $ab \neq ba$. The Matlab/Octave function shown in Listing 1 accepts two quaternions p and q, both represented by 4D-vectors, and computes pq.

Listing 1: Computing pq for two quaternions p and q. Note that the operation is not commutative.

```
function pq = qmul( p, q )
  a0 = p(1); a = p(2:4);
  b0 = q(1); b = q(2:4);
  acb = a0*b + b0*a + cross(a,b);
  pq = [a0*b0-dot(a,b); a0*b + b0*a + cross(a,b)];
end
```

Like for complex numbers, a quaternion $q = (q_0, \mathbf{q})$ does have a *conjugate* quaternion q^c which is defined by $q^c = (q_0, -\mathbf{q})$. The conjugate now enables the computation of the norm $|q|$ of a quaternion, which is defined by $|q|^2 = qq^c$. Note that quaternions u with length $|u|^2 = uu^c = 1$ are called *unit* quaternions.

An important subclass of quaternions is given by *pure* quaternions $q = (0, \mathbf{q})$, which are defined to have a zero real part. For pure quaternions, the operations (1) are simplified accordingly.

By using the rules of (1), quaternions now can be used for computing rotations in a 3-dimensional Euclidean space. Each vector $\mathbf{r} = (r_1, r_2, r_3)'$ of the space is represented by a pure quaternion $r = (0, \mathbf{r})$. Rotations in the space then can be characterized by a unit quaternion u by computing

$$\hat{r} = uru^c. \tag{2}$$

Note that $\hat{r} = (0, \hat{\mathbf{r}})$ is again a pure quaternion, whose vector part $\hat{\mathbf{r}}$ equals $\mathbf{r}$ rotated by some angle ϕ and using the rotation axis $\mathbf{u}$, i.e., the vector part of u. The Matlab/Octave function shown in Listing 2 rotates a vector $p = (0, \mathbf{p})$ by the quaternion u.

Listing 2: Rotating vector $\mathbf{p}$ ($p = (0, \mathbf{p})'$) by a rotation represented by a unit quaternion u. The result is again a pure quaternion holding the rotated vector in its vector part.

```
function upuc = rotquat( p, u )
  uc = [u(1); -u(2:4)];
  up = qmul(u,p);
  upuc = qmul(up,uc);
end
```

Given a desired rotation axis $\mathbf{a} = (a_1, a_2, a_3)'$ and a rotation angle ϕ, the quaternion u representing this rotation is constructed by

$$u = (cos\frac{\phi}{2}, sin\frac{\phi}{2}\frac{\mathbf{a}}{|a|}). \tag{3}$$

Thus, given this quaternion, using (2) rotates any desired vector $\mathbf{r}$ by angle ϕ and axis $\mathbf{a}$. The Matlab/Octave function shown in Listing 3 constructs a rotation quaternion u from a rotation angle ϕ and a 3D-vector describing the rotation axis.

Listing 3: Computing a rotation quaternion u.

```
function u = quat( phi, a )
  u = [cos(phi/2); sin(phi/2)*normc([a(1); a(2); a(3)])];
end
```

5 The Optimal Rotation

In linear algebra, rotations are represented by orthonormal square matrices U, rotating a vector $\mathbf{x}$ is then achieved by multiplying it from the right: $\hat{\mathbf{x}} = U\mathbf{x}$. Given two sets of vectors $\{\mathbf{x}_k\}$ and $\{\mathbf{y}_k\}$, Coutsias et al. (2004) have shown how to compute an optimal rotation U (through the use of quaternions) such that the overall error

$$E := \frac{1}{N}\sum_{k=1}^{N} |U\mathbf{x}_k - \mathbf{y}_k|^2 \tag{4}$$

is minimized. The respective Matlab/Octave function is shown in Listing 4.

Centering both sets by computing the mean vectors $\bar{\mathbf{x}}$ and $\bar{\mathbf{y}}$ of each set, and then subtracting $\bar{\mathbf{x}}$ from each vector of $\{\mathbf{x}_k\}$ and $\bar{\mathbf{y}}$ from each vector of $\{\mathbf{y}_k\}$ will give lower errors but is not necessary for the method to work. Specifically for acceleration time series, subtracting the mean (in practice a moving average computed over sliding time windows) removes the static offset caused by gravity and has previously also been found advantageous for comparing magnitudes by Mayrhofer and Gellersen (2009). Like in (Coutsias et al., 2004) it is furthermore assumed that X and Y are $3 \times N$ matrices, whose columns hold the vectors $\mathbf{x}_k$ and $\mathbf{y}_k$.

Listing 4: Computing the optimal rotation u given vectors $\mathbf{x}_k$ and $\mathbf{y}_k$.

```
function [E,P] = residuum( X, Y )
  %correlation matrix
  R=X*(Y');
  %matrix holding all rotations
  F=[R(1,1)+R(2,2)+R(3,3), R(2,3)-R(3,2), R(3,1)-R(1,3),
      R(1,2)-R(2,1);
       R(2,3)-R(3,2), R(1,1)-R(2,2)-R(3,3), R(1,2)+R(2,1)
           , R(1,3)+R(3,1);
       R(3,1)-R(1,3), R(1,2)+R(2,1), -R(1,1)+R(2,2)-R
           (3,3), R(2,3)+R(3,2);
       R(1,2)-R(2,1), R(1,3)+R(3,1), R(2,3)+R(3,2), -R
           (1,1)-R(2,2)+R(3,3) ];
  %compute eigenvector evvmax of largest eigenvalue ev
  [V,D] = eig(F);
  ev = D(1,1); evvmax = V(:,1); %first ev and evvmax
  for i=2:4
         if D(i,i)>ev
                ev = D(i,i); evvmax = V(:,i); %remember
                    largest ev and evvmax
         end
  end
  [E,P]=reser(X,Y,evvmax); %compute error and optimal P=
      UX
end
```

Listing 5: Computing the optimal predictor P and the error E.

```
function [E,P] = reser( X, Y, u )
  [n,m] = size(X);
  E=0; P=[];
  for k=1:m
       y = rotquat([0; X(k,:)],u);     %rotate the x
           vectors
       P=[P y(2:4)']; %store in P=UX
       E = E + norm( Y(:,k)~-~P(:,k) )$^\wedge$2;   %sum
           of norms of differences
  end
  E = E/m;
end
```

The Matlab/Octave function `reser` shown in Listing 5 shows how to compute the error E given by (4) and the optimal predictor $P = UX$. Note that using the largest eigenvalue of the matrix F directly, as proposed by Coutsias et al. (2004), frequently results in complex values due to roundoff errors and negative values inside the root operation (see definition of e_q in (Coutsias et al., 2004)), which mathematically is impossible. Thus we recommend to directly compute E by using (4).

6 Evaluation

To evaluate our approach of determining the spatial alignment of 3D acceleration sensor and derotating time series before doing comparisons we apply it to real world acceleration data. We use acceleration time series recorded pairwise by shaking two 3D accelerometers together and estimate if two devices were shaken together based on various similarity measures on their time series. In order to quantify the gain of derotating time series before comparing them we separately compare pairwise time series without and with derotation. Similarity between individual axes of two 3D accelerometers strongly depends on the spatial alignment of the accelerometers — therefore, comparing the original (arbitrarily aligned) axes directly with each other cannot be expected to yield useful results. Consequently, when not derotating time series, we apply the well known practice of computing and comparing the magnitude time series instead of utilizing individual axes. Before calculating these magnitudes we compensate for gravity by normalizing time series and subtracting their mean (to discard the influence of gravity) for each axis and device. To demonstrate that derotating time series is possible even with short recordings, we limit all time series to a duration of 2 s, which seems a compromise between distinguishability and usability for the user-mediated device pairing problem (cf. (Mayrhofer and Gellersen, 2009)).

To compare two acceleration time series we utilize well known approaches – indicating either the amount of divergence (error) or similarity. As distance metrics indicating error in the time domain we use: root mean squared error (RMSE), mean absolute error (MAE), median absolute error (median), standard deviation of errors (SD), median absolute deviation of errors (MAD) and dynamic time warping (DTW). As distance metrics indicating error in the frequency domain we use: RMSE of the FFT power spectra of both time series (FFT RMSE), MAE of the FFT power spectra (FFT MAE), median absolute error of the FFT power spectra (FFT median), standard deviation of errors of the FFT power spectra (FFT SD) and median absolute deviation errors of the FFT power spectra (FFT MAD)[1]. As metrics indicating similarity we use correlation coefficients by Pearson (1895) (Product-Moment Correlation Coefficient), Spearman (1904) (Rank Correlation Coefficient), and Kendall (1938) (Tau Rank Correlation Coefficient) as well as magnitude squared coherence (coherence), which has been used frequently on comparing acceleration time series in previous research (Ben-Pazi et al., 2001; Cornelius and Kotz, 2012; Dargie, 2009; Findling et al., 2014; Lester et al., 2004; Mayrhofer and Gellersen, 2009).

[1]In preliminary evaluations, errors of FFT phase information have been evaluated as well, which seemed to not yield feasible distinguishing information and consequently have been excluded in this evaluation.

For coherence we apply parametrization as stated by Mayrhofer and Gellersen (2009). We expect coherence – representing the most sophisticated amongst the selected approaches – to yield better results than correlation coefficients, which we further expect to yield better results than the selected error based metrics.

6.1 Evaluation data

As source of acceleration time series we use the u'smile ShakeUnlock database[2] published by Findling et al. (2014). It contains pairwise 3D acceleration time series of two devices shaken together: a mobile phone held in the hand and a watch strapped to the wrist (Figure 1), shaken for about 10 s. In total, the database contains 29 participants shaking two devices 20 times, which results in 580 records with two 3D acceleration time series in each record. Acceleration has been recorded with 100 Hz across all devices. As we limit time series to a duration of 2 s we therefore only utilize 200 values per time series for comparison.

Summarizing, we use 696000 acceleration sensor values (580 records of 2 time series with 200 samples in 3 dimensions) from this database for evaluating our derotation approach.

(a) Front side (b) Rear side

Figure 1: Acceleration time series recording setup with the u'smile ShakeUnlock database (picture taken from (Findling et al., 2014)).

6.2 Time series derotation example

Figure 2 shows two sample magnitude time series from the u'smile ShakeUnlock database. The samples were originated by two devices actually shaken together. Axes have been gravity adjusted before calculating the magnitudes, and the time series have been limited to a duration of 2 s. Although magnitudes are not equal, their similarity is obvious: phasing is similar, though overall the amplitude seems to be higher for device 2.

Looking at time series of individual axes for the same samples, similarity is not as obvious anymore (Figure 3). Although acceleration phasings and amplitudes are similar for axis 1, for axis 2 only phasings are obviously similar –

[2]Evaluation data fetched from `http://usmile.at/downloads`.

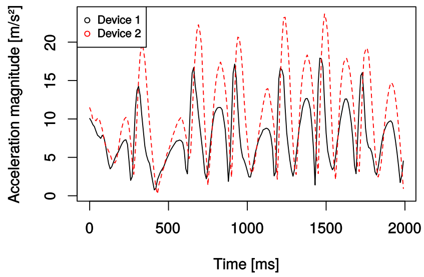


Figure 2: Sample acceleration time series magnitudes of device 1 and 2.

for axis 3 there is no obviously visible similarity. After applying derotation (by rotating the 3D acceleration time series of device 1 according to the spatial alignment of device 2) similarity is obvious again for all axes.

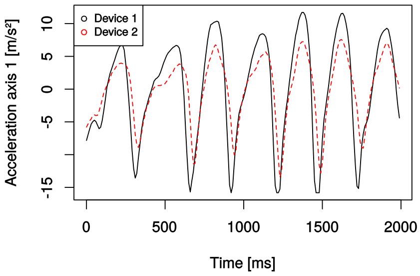
Table 1 provides previously stated metrics for these two sample time series – for comparing magnitudes and individual axes, without and with applying derotation. As expected, comparing not-rotated time series of individual axes causes highest errors/least similarities. Comparing magnitudes causes smaller errors/higher similarities. Overall, smallest errors/highest similarities were achieved by comparing derotated, individual axes.

		Without derotation			With derotation		
Metric	Mag.	A1	A2	A3	A1	A2	A3
RMSE	5.65	4.25	6.71	8.39	3.24	4.16	4.49
MAE	4.44	3.23	5.69	7.34	2.28	3.52	3.67
MEDIAN	3.88	2.71	5.15	7.71	1.38	3.56	3.24
SD	6.12	5.31	6.96	8.14	3.63	5.63	5.58
MAD	5.39	2.20	5.35	6.93	1.65	4.46	3.07
Pearson	0.56	0.87	0.87	-0.57	0.77	0.91	0.74
Kendall	0.44	0.76	0.69	-0.49	0.68	0.77	0.60
Spearman	0.61	0.92	0.88	-0.68	0.86	0.92	0.80
DTW	1.25	1.03	2.62	2.38	0.61	1.06	0.71
FFT power RMSE	69.07	40.83	91.56	68.29	27.09	40.41	30.90
FFT power MAE	22.04	14.52	24.75	19.97	11.36	13.40	10.84
FFT power Median	7.09	3.34	2.93	4.03	2.73	3.40	3.18
FFT power SD	67.13	39.26	89.09	66.61	26.61	39.65	30.52
FFT power MAD	8.26	3.38	2.59	3.01	2.42	3.33	2.34
Coherence	0.80	0.61	0.56	0.60	0.63	0.65	0.64

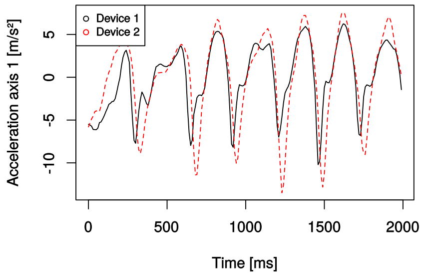
Table 1: Similarity metrics of sample acceleration time series, for magnitudes (Mag.) and individual axes (A1-A3), without and with derotating axes before comparison.

6.3 Experimental setup

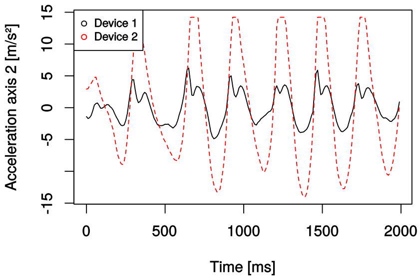
As the effect of increasing similarity between two 3D time series applies for correlated time series (devices shaken together) as well as for uncorrelated time series (devices not shaken together) an evaluation must include both correlated and uncorrelated time series. For this reason we determine if devices were shaken together by comparing all possible combinations of acceleration time series from the database. Each comparison results in a single, scalar metric value s. Applying a threshold t so that $\min(s) \leq t \leq \max(s)$ to s we obtain the true match rate (TMR, rate of devices correctly being identified as shaken



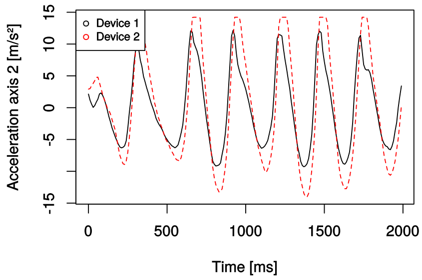
(a) Axis 1 without derotation



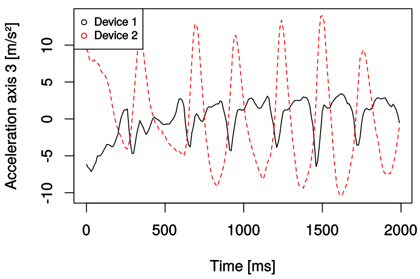
(b) Axis 1 with derotation



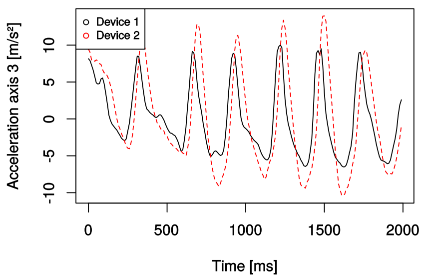
(c) Axis 2 without derotation



(d) Axis 2 with derotation



(e) Axis 3 without derotation



(f) Axis 3 with derotation

Figure 3: Sample 3D acceleration time series axes without (a, c, e) and with derotation (b, d, f).

together) and the true non match rate (TNMR, rate of devices correctly being identified as not shaken together). The false match rate (FMR, rate of devices incorrectly being identified as shaken together) is the complement to the TNMR: FMR = 1 − TNMR. For error based metric values (RMSE, MAE, MEDIAN), if $s < t$ time series are identified as shaken together – if $s \geq t$ time series are identified as not shaken together. Respectively, for similarity based metric values (correlation coefficient, coherence), if $s < t$ time series are identified as not shaken together – if $s \geq t$ time series are identified as shaken together. Comparison with ground truth (if the two time series have been recorded from devices actually shaken together) originates the TMR and TNMR (resp. FMR) used in the receiver operating characteristic (ROC) curves. To obtain the TMR we perform a pairwise comparison of all 580 pairs of acceleration time series (watch and phone) from the database. To obtain the TNMR (resp. FMR) we use all $1160 \cdot 1159/2/2 = 336110$ other possible pairwise comparisons of time series from a phone and watch each[3].

6.4 Results

Evaluation results clearly show that derotating and comparing individual axes of acceleration time series (Figure 4b) yields better results than computing and comparing their magnitudes (Figure 4a). This supports our hypothesis that derotating pairwise 3D acceleration time series before doing comparisons improves comparison results. Table 2 states the equal error rate EER $= 1 - \text{TMR} \simeq 1 - \text{TNMR}$ and the square root of the minimum squared error rate $\sqrt{\text{MSER}} = \sqrt{\min((1-\text{TMR})^2 + (1-\text{TNMR})^2)}$ for comparing time series based on magnitudes and on derotated, individual axes for all metrics[4].

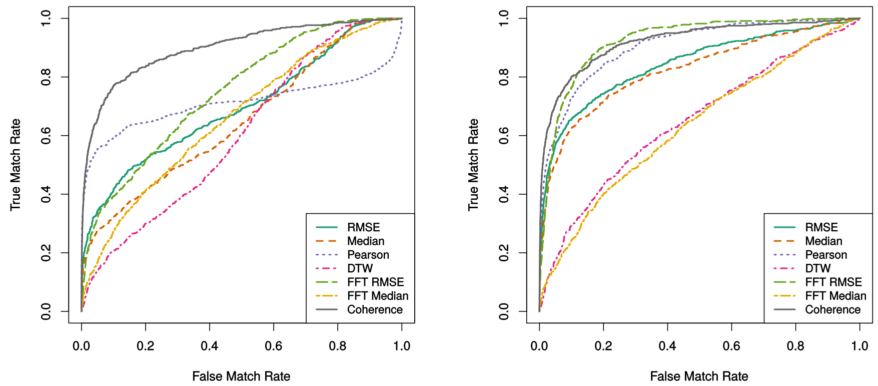
	Time series magnitudes					Derotated axes time series				
	EER	$\sqrt{\text{MSER}}$	$\sqrt{\text{MSER}}$ TPR	$\sqrt{\text{MSER}}$ TNR	AUC	EER	$\sqrt{\text{MSER}}$	$\sqrt{\text{MSER}}$ TPR	$\sqrt{\text{MSER}}$ TNR	AUC
RMSE	0.373	0.512	0.544	0.768	0.696	0.233	0.322	0.741	0.808	0.844
MAE	0.391	0.528	0.548	0.728	0.682	0.235	0.324	0.74	0.808	0.841
Median	0.429	0.587	0.485	0.72	0.65	0.246	0.34	0.738	0.784	0.824
SD	0.354	0.462	0.587	0.792	0.698	0.233	0.322	0.741	0.808	0.844
MAD	0.355	0.473	0.603	0.742	0.687	0.235	0.324	0.74	0.808	0.841
Pearson	0.318	0.394	0.637	0.845	0.711	0.18	0.252	0.817	0.827	0.905
Spearman	0.318	0.391	0.639	0.85	0.712	0.181	0.254	0.808	0.834	0.904
Kendall	0.316	0.393	0.646	0.828	0.712	0.181	0.253	0.833	0.81	0.904
DTW	0.458	0.629	0.666	0.466	0.605	0.391	0.545	0.56	0.678	0.649
FFT RMSE	0.33	0.466	0.675	0.665	0.749	0.149	0.206	0.874	0.836	0.923
FFT MAE	0.189	0.264	0.819	0.809	0.887	0.214	0.301	0.788	0.786	0.863
FFT median	0.395	0.554	0.591	0.627	0.661	0.411	0.575	0.56	0.629	0.632
FFT SD	0.327	0.462	0.671	0.675	0.752	0.149	0.205	0.88	0.834	0.925
FFT MAD	0.239	0.325	0.734	0.813	0.844	0.208	0.291	0.808	0.782	0.867
Coherence	0.178	0.241	0.815	0.845	0.899	0.156	0.217	0.856	0.837	0.921

Table 2: Evaluation results: axes derotation decreases error rates for deciding if devices have been shaken together.

As expected, overall coherence based comparison yields good results, followed by correlation coefficients and less sophisticated error based metrics. Although results for some error based metrics are close to random when based on magnitudes, there is a significant improvement when derotating and comparing individual axes instead – e.g. FFT based RMSE delivers results similar to coherence. Interestingly, for correlation coefficient based metrics derotating

[3] Assuming that comparing time series is commutative, namely comparing time series A with B yields the same results as B with A – which applies to all our metrics.

[4] $\sqrt{\text{MSER}}$ represents the Euclidean distance between the point TMR = TNMR = 1 and the resulting TMR/FMR closest to this point.



(a) Magnitude based comparisons

(b) Derotated individual axis based comparisons

Figure 4: Selected performance metrics for deciding if devices have been shaken together using their acceleration time series, based on comparing magnitudes and derotated, individual time series.

and comparing single axes significantly decreases the FMR for high TMR. The close-to-flat area in the magnitude ROC curve indicates, that – for the data used in our evaluation – magnitude correlation coefficient based separation without significantly increasing FMR is possible for about 70-80% of samples. Separation of the remaining 20-30% is either erroneous or causes a significant rise in FMR. This effect disappears when derotating and comparing individual axes.

7 Conclusions

We have contributed a method for determining relative spatial alignment of devices based on independently recorded acceleration time series during common movement. Using quaternions, our approach allows to *analytically* compute the optimal rotation between the respective reference systems with a run-time complexity of $O(N^2)$ for N samples. The significant advantage over heuristic approaches is that this method is guaranteed to provide the optimal rotation with deterministic run-time. We suggest that this approach is beneficial ofr all applications comparing acceleration (or other 3D sensors) time series that were recorded independently with potentially arbitrary and unknown alignment, and that it can be used on systems with limited computational resources such as mobile phones.

Using real world experimental data and coherence as the currently best performing distance metric for determining if two devices were shaken together, we see an improvement of about 11% in equal error rate by derotating the coordinate system of one of the devices before comparison. We note that this is the approach taken in previous research, relying on magnitude only and discarding angular information of all movement, and it still benefits from applying the method proposed in this paper.

We suspect that other methods for comparing 3D time series and using this additional information – which was previously impossible with arbitrarily rotated devices – can achieve significantly lower error rates. Our proposed analytical derotation method therefore opens new research questions for future work.

All Matlab/Octave scripts and data sets are available under the terms of the GNU Lesser General Public License (LGPL) at `http://usmile.at/downloads`.

Acknowledgments

This work has been carried out within the scope of *u'smile*, the Josef Ressel Center for User-Friendly Secure Mobile Environments, funded by the Christian Doppler Gesellschaft, A1 Telekom Austria AG, Drei-Banken-EDV GmbH, LG Nexera Business Solutions AG, NXP Semiconductors Austria GmbH, and Österreichische Staatsdruckerei GmbH.

References

Ben-Pazi, H., Bergman, H., Goldberg, J. A., Giladi, N., Hansel, D., Reches, A., and Simon, E. S. (2001). Synchrony of rest tremor in multiple limbs in parkinson's disease: evidence for multiple oscillators. *Journal of Neural Transmission*, 108(3):287–296.

Bichler, D., Stromberg, G., Huemer, M., and Löw, M. (2007). Key generation based on acceleration data of shaking processes. In *Proc. UbiComp 2007*, volume 4717 of *LNCS*, pages 304–317. Springer-Verlag.

Choe, S. B. and Faraway, J. J. (2004). Modeling head and hand orientation during motion using quaternions. Technical Report 2004-01-2179.

Cornelius, C. T. and Kotz, D. F. (2012). Recognizing whether sensors are on the same body. *Pervasive and Mobile Computing*, 8(6):822–836. Special Issue on Pervasive Healthcare.

Coutsias, E. A., Seok, C., and Dill, K. A. (2004). Using quaternions to calculate rmsd. *Wiley InterScience*.

Dargie, W. (2009). Analysis of time and frequency domain features of accelerometer measurements. In *Computer Communications and Networks, 2009. ICCN 2009. Proc. of 18th Internatonal Conference on*, pages 1–6.

Findling, R. D., Muaaz, M., Hintze, D., and Mayrhofer, R. (2014). ShakeUnlock: Securely unlock mobile devices by shaking them together. In *Proc. MoMM 2014: 12th International Conference on Advances in Mobile Computing and Multimedia*, pages 165–174, New York, NY, USA. ACM Press. *Awarded MoMM 2014 best paper.*

Groza, B. and Mayrhofer, R. (2012). SAPHE - simple accelerometer based wireless pairing with heuristic trees. In *Proc. MoMM 2012: 10th International Conference on Advances in Mobile Computing and Multimedia*, pages 161–168, New York, NY, USA. ACM Press.

Kendall, M. (1938). A new measure of rank correlation. *Biometrika*.

Kirovski, D., Sinclair, M., and Wilson, D. (2007). The Martini Synch. Technical Report MSR-TR-2007-123, Microsoft Research.

Kuipers, J. B. (2002). *Quaternions and Rotation Sequences: A Primer with Applications to Orbits, Aerospace and Virtual Reality*. Princeton Univers. Press.

Kunze, K. and Lukowicz, P. (2008). Dealing with sensor displacement in motion-based onbody activity recognition systems. In *Proc. Ubicomp 2008*, pages 20–29. ACM Press.

Lester, J., Hannaford, B., and Borriello, G. (2004). "Are you with me?" – Using accelerometers to determine if two devices are carried by the same person. In *Proc. Pervasive 2004*, pages 33–50. Springer-Verlag.

Marin-Perianu, R., Marin-Perianu, M., Havinga, P., and Scholten, H. (2007). Movement-based group awareness with wireless sensor networks. In *Proc. Pervasive 2007*, pages 298–315. Springer-Verlag.

Mayrhofer, R. and Gellersen, H. (2009). Shake well before use: Intuitive and secure pairing of mobile devices. *Mobile Computing, IEEE Transactions on*, 8(6):792–806.

Pearson, K. (1895). Note on regression and inheritance in the case of two parents. In *Proc. of the Royal Society of London*, volume 58, pages 240–242.

Spearman, C. (1904). The proof and measurement of association between two rings. *American Journal of Psychology*, 15:72–101.